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cf. I'm absent during 7-16 nov. for a conference.
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Proposition: MC & DC

Martingale process in *Progressive Quenching*

Background — Martingale

A *stochastic process* $\{m_1, m_2, \dots, m_n\}$ is *martingale* with respect to the *stochastic process* $\{X_1, X_2, \dots, X_n\}$

$\leftrightarrow$ $E[\, m_{t+1} \mid \{X_1, X_2, \dots, X_n\}\,] = m_t$ $E[x \mid y]$: conditional expectation

An example : $m = x$ = random walk

Martingale is one of the central concepts in the probability theory. **BUT** in **physics**, its meaning & importance beyond random walk was little known until very recently.

Recent major discoveries

1. So-called ***fluctuation theorems*** *(there are many versions) are recognized to be exponential martingale of the "entropy production".* (Cheterite & Gupta: 2011, Roldan *et al., 2017)*

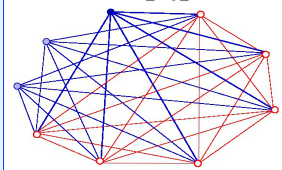
2. In the protocol of **progressive quenching** (see right →) non-trivial martingale emerges. (Ventéjou-KS, Etienne-KS: 2018)

More about stochastic processes → **M2 Course in 2nd semester (Ken Sekimoto)**

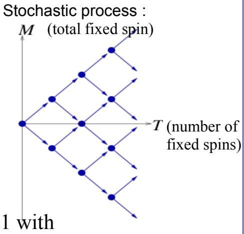
Example (Ventéjou & KS, 2018)

Ising spin model on a *complete lattice* .

$$\beta H_{N,J,h} = -\beta J \sum_{1\le i<j\le N} s_i s_j - \beta h \sum_{1\le i\le N} s_i \qquad \beta = 1, \quad J = \frac{j_0}{N_0}$$



Progressive Quenching:
(t:q)-step: quench *t*-th spin
(t:r)-step: equilibrate (*N*–*t*) spins
(t+1:q)-step: … repeat until *N* fixed



At each *(t:*q)-step, *t* -th spin s_t takes ± 1 with $\langle s_t \rangle = m_t$: equilibrium mean spin → **martingale**

Utility of martingale ~ *Stochastic conservation* *(Memory is kept & updated.)*

(1) Statistical ensemble of the *final* quenched magnetization : $M_N = \sum_{t=1}^{N} s_t$
(2) Information : " M_T is common " ($1 < T < N$)
(3) Martingale "optional stopping" theorem

→ Inference of M_T value up to O(1) with ~N^2 steps. (not ~exp(cN))

Proposition of Master training ("stage")

Object: How is the breaking of *martingale* associated to space/time structure ?

Model studied : 1D spin systems, ***p***-nn spin coupling, ***q***-spin simultaneous quenching
Known fact: Progressive quenching of 1nn & 2nn systems (nn = nearest neighbor)

Exploitation :

New : 1D systems with ***long range interaction*** : $J_{i,j} \sim |i-j|$^$(-q)$ → martingale ?   **Method** : Transfer matrix method
New : 1D systems with quenching at ***finite rate*** → martingale ?   **Method** : Glauber dynamics

Preparation for DC :
Martingale and other advanced stochastic methods.
Information stochastic thermodynamics.
Network dynamics (cavity, random network, first passage), etc. → **M2 Course in 2nd semester (Ken Sekimoto)**

Proposition of PhD thesis

(Conceptual)
Martingale property as *stochastic conservation in Physics*

— ***Infortmation thermodynamic approach***

* **Progressive Quenching**
= measuring partial information of system
+ measurement-dependent feed-back control

* **Particularity of Progressive Quenching**
→ **System size / Degrees of freedom are time dependent**

— ***Symmetry principle of stochastic conservation***

(Theoretical+numerical)
Martingale property on networks under Progressive Quenching

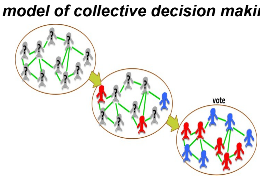
— ***On Bethe tree, factor graph*** **(cavity approx'n)**
→ (Breaking of) martingale vs coordination nb

* **Progressive Quenching**
= isolating branches
+ freezing effective field

— **On complete network +** ***thermalization at finite rate***
→ (Breaking of) martingale /inform'n analysys

(Application)
Progressive Quenching in Sociological / Neurological contextes

— ***model of collective decision making*** **(e.g. Brexit)**



— ***model of knowledge network formation***

B. Ventéjou, K. Sekimoto; Phys. Rev. E **97**, 062150 (2018)
M. Etienne, K. Sekimoto; Acta Physica Polonica, **49**, 883-892 (2018)
Stochastic Energetics, K. Sekimoto (Springer, Lecture Notes in Physics **799** (2010))

Read also, R. Chetrite and S. Gupta, J. Stat. Phys. 143, 543 (2011).
I. Neri, E. Roldán, and F. Jülicher, Phys. Rev. X 7, 011019 (2017).