General covariance principle for discrete time quantum systems

Master internship project (M2)

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Keywords: *general covariance principle, quantum cellular automata, curved dirac equation, discret space-time*

Place: Laboratoire d'Informatique et Systemes (LIF), Natural Computing team (CaNa). Scientific environment: The CaNa research group (Pablo Arrighi, Giuseppe Di Molfetta, Kevin Perrot, Enrico Porreca, Sylvain Sen) seeks to capture at the formal level some of the fundamental paradigms of theoretical physics and biology, via the models and approaches of theoretical computer science and discrete mathematics. The group is located in Luminy, Marseille, France, and benefits from a rich scientific environment with the Cellular Automata experts of I2M (Pierre Guillon, Guillaume Theyssier) and the physicists from CPT (Alberto Verga, Thomas Krajewski).

Supervisor: Giuseppe Di Molfetta (GDM) obtained his PhD in Quantum Information at UPMC and ENS in Paris in 2015, since then he held prestigious positions as JSPS fellow at the National Institute of Natural Science in Japan, Severo Ochoa Research Junior at IFIC in Spain and visiting scientist at Perimeter Institute. He is now Maître de Conferences in the CaNa group (Calcul Naturel) of the LIS (Laboratoire d'Informatique et Systemes) at Aix-Marseille University (AMU), but still co-supervises a PhD thesis in Valencia. He already supervised several master students in Japan, France and Spain. His main domain of expertise is quantum information theory and quantum simulation on lattice, including relativistic effects. He has national (Marseille, Paris, Grenoble) and international collaborations (Brazil, Japan, Germany, UK), resulting in already 20 publications since 2013 in high-impact international journals (eg. New Journal of Physics, PRA, PRE). Profil: Google Scholar

Theme and goals The notion of observer is crucial both in Quantum Mechanics (QM), as a measurer, and in General Relativity, as a reference frame. After all "General Relativity" (GR) means but observer-independence, covariance. But what does covariance mean, in quantum mechanics? Covariance in QM may be labelled a premature question, since there is no widely accepted common mathematical grounds between GR and QM. Still, some proposals are trending, that are often based on discrete structures (graphs, superpositions of them)[1]. Yet these seem to make the matter worse, as discreteness/quantification ought to break covariance.

Our approach to this matter comes from Quantum Computing à la Feynman, i.e. Quantum Simulation. Quantum Walks are reminiscent of the Feynman checkerboard model of the electron, they have been proved to converge towards the Dirac Equation[2], they therefore describe a quantum circuit that simulates the electron's propagation. The Dirac Equation is Lorentz-covariant, what about the Dirac QW?

Quite often, different circuits end up behaving in exactly the same manner, and quantum circuits are no exceptions to that. In fact, much of the phenomenology of QM can be recovered by assuming that a handful of quantum circuit equivalences hold true[3]. We have developed a small, diagrammatic theory of equivalence of quantum circuits, which transforms a Dirac QW into another, in the same way a Dirac Equation transforms into another with a change of reference [4]. In the continuum limit the former transform leads to the latter; we have characterized Lorentz-covariance in the form of a circuit-equivalence.

Lately, we have shown that Quantum Walks can also simulate the Curved Dirac Equation [5] The Curved Dirac Equation is one of the very few established pieces of merged GR and QM. It is fully covariant. Hence, what about the Covariance of Curved Dirac QW? Can it be understood as a more liberal form of circuit-equivalence?

Perspectives: The natural extension of this project is a theoretical thesis. Such a PhD project could bring together two teams of CPT and LIS, to explore at the frontier between quantum information and theoretical physics, especially in the perspective of simulating quantum gravity effects. The internship will likely benefit from an international consortium, built recently with the also the joint effort of the CPT (quantum gravity group) and the LIS (CANA group).

The internship will be funded as legally required.

Prerequisite: Quantum Mechanics, General Relativity, Quantum Field Theory

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