Growing Complex Networks via quantum rules

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Place: Laboratoire d'Informatique et Systemes (LIF), Natural Computing team (CaNa). Scientific environment: The CaNa research group (Pablo Arrighi, Giuseppe Di Molfetta, Kevin Perrot, Enrico Porreca, Sylvain Sen) seeks to capture at the formal level some of the fundamental paradigms of theoretical physics and biology, via the models and approaches of theoretical computer science and discrete mathematics. The group is located in Luminy, Marseille, France, and benefits from a rich scientific environment with the Cellular Automata experts of I2M (Pierre Guillon, Guillaume Theyssier) and the physicists from CPT (Alberto Verga, Thomas Krajewski). *This specific project will also benefits from an existing scientific collaboration with Dr. Filippo Miatto at Telecom ParisTech, LTCI.*

Supervisor: Giuseppe Di Molfetta (GDM) obtained his PhD in Quantum Information at UPMC and ENS in Paris in 2015, since then he held prestigious positions as JSPS fellow at the National Institute of Natural Science in Japan, Severo Ochoa Research Junior at IFIC in Spain and visiting scientist at Perimeter Institute. He is now Maître de Conferences in the CaNa group (Calcul Naturel) of the LIS (Laboratoire d'Informatique et Systemes) at Aix-Marseille University (AMU), but still co-supervises a PhD thesis in Valencia. He already supervised several master students in Japan, France and Spain. His main domain of expertise is quantum information theory and quantum simulation on lattice, including relativistic effects. He has national (Marseille, Paris, Grenoble) and international collaborations (Brazil, Japan, Germany, UK), resulting in already 20 publications since 2013 in high-impact international journals (eg. New Journal of Physics, PRA, PRE). Profil: Google Scholar

Theme and goals Complex networks are discrete structures that model a large variety of real systems, from natural to socioeconomic phenomena, such as the protein-protein interaction, the brain, the internet and the international trading. This project merges the field of complex networks (CNs) with the most recent developments in quantum information (QI); two of the raising forces of the information era. Quantum CNs (QCNs), are networks where each vertex and possible link is associated with a quantum state. The dynamics over them is unitary, following the standard quantum mechanical postulates. The main motivation to understand and model QCNs, as opposed to studying regular quantum lattices, is at least threefold : (i) non-trivial topologies possesses useful properties such as a great robustness to random errors[1]; (ii) recently experimental and theoretical researchers have understood the importance of QCNs to model the discrete dynamics and geometrical structure of natural phenomena at quantum scale, such as protein folding in DNA[2], light energy harvesting in photosynthesis and even emergent quantum gravity[3], and (iii) foreseeing the quantum web[4]. Although recent, there is a variety of theoretical studies on QCNs, but almost none on their dynamical aspects and their growth. The core idea of this proposal is to contribute to fulfill this theoretical gap and focus on growing QCNs (GQCNs). Our research project makes key contributions to three big questions: How GQCNs can be mathematically modeled? Which are the quantum counterparts of the well-known classical phenomena in large CNs, such as percolation, clustering or small-world topologies? All these questions spread across the foundations of computer sciences and mathematics and have as final goal to refine and implement general mathematical results on several specific model of GQCNs.

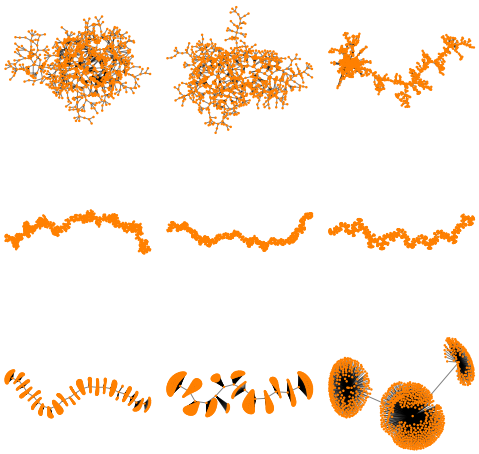

FIG. 1: Random networks with 1000 links grown with identical rules, except for the parameter of the exponential distribution of the measurement time. First row: λ =0.01, 0.1, 0.5, second row: λ =1, 5, 10, bottom row: λ =50, 100, 500.

More in detail.. In the first and main phase of the project we grow a particular kind of Markovian network, a quantum tree, through quantum evolution and quantum measurements. We allow only creation of nodes and links. At each instant in time the quantum graph state $|G\rangle$ is represented by the adjacency matrix $A(G)$. Then, a quantum walker state $|W(t)\rangle \in \mathbb{C}^{|G|}$, at time t, propagates in continuous time on the graph, via the well know unitary evolution $U(t) = \exp(iA(G)t)$. Now instead to keep fix the adjacency matrix, we measure the position of the walker at time intervals distributed according to some probability density $p(t)$, *e.g.*, we choose the exponential distribution to model the measurement events as a Poisson point process. Because of the measurement, the walker will collapse into a random node, distributed on the graph according to the walker's state. On the node where it collapsed we attach a new node through a single link, then the quantum walk evolution restarts from there. This process can be thought of a measurement from the perspective of the new node, that probes the quantum state on the graph, causing it to collapse, and then it becomes "part" of it.

Let me stress that we have two kind of time parameters: one is related to the discrete evolution of the quantum graph state $|G\rangle$, the other is continuous and it is the evolution time of the quantum walker. This can been also seen as a limit in which the time scale of the environment interaction with the quantum graph state is much smaller respect the time scale for growing the network. By evolving graphs with the rules above, we obtain a random graph each time. What is interesting is that the exponential distribution parameter does not control the "scale" of the graph, but rather it varies drastically its *topology*. We evolved each graph below for 1000 steps, ending up with 1001 nodes each (see Figure). For all the graphs produced with this rule, the probability $P(k)$ for a random node to have k connections follows a power law, which is indicative of a scale-free network.

We would like to investigate numerically and analytically this topological phase transition. *In a second part we will be interested in generalizing our basic quantum rules for growing complex network to non-vanishing clusters.*

Perspectives: The natural extension of this project is a theoretical thesis. Such a PhD project could bring together two teams of CPT and LIS and one at ParisTech, to explore at the frontier between quantum information and theoretical physics, the basic mechanisms relating network complexity, quantum states and dynamics. We will investigate interacting many-body quantum walks and automata on dynamical graphs, and use their entanglement evolution and other information tools to link topological and information processing properties. We are interested in (i) the mathematical general description of quantum complex network dynamics, (ii) the possibility to define in these systems a causal structure, and finally (ii) the analysis of eventual quantum transitions induced by changes in the growing network underlying topology (iv) the mechanism of relaxation and thermalization of isolated systems. The interplay of many-body effects due to local interactions and the evolving graph topology, is largely unexplored. We want, in particular, to determine the conditions for the emergence of macroscopic well defined properties (a well defined magnetic phase, a particular geometry in gravitation, a non classical resource in quantum computing) in a microscopic chaotic system, essentially controlled by the entanglement growth. We think that the basic relation between information and physics laws, is rich enough to trigger a scientific activity in Marseille, which may contribute to the formation of a larger community working in this emerging discipline.

The internship will be funded as legally required.

Prerequisite: Strong programming skills, Quantum Mechanics and/or Statistical Physics background is an atout

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