M2 Internship and PhD Project

Nonperturbative renormalization group approach to disordered polymerized membranes

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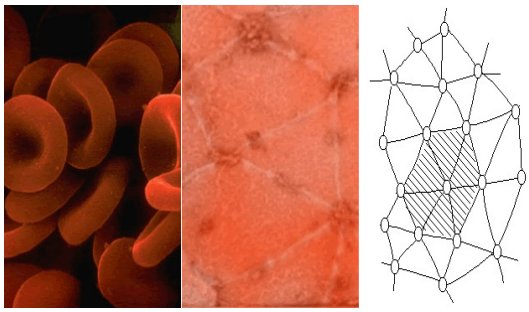
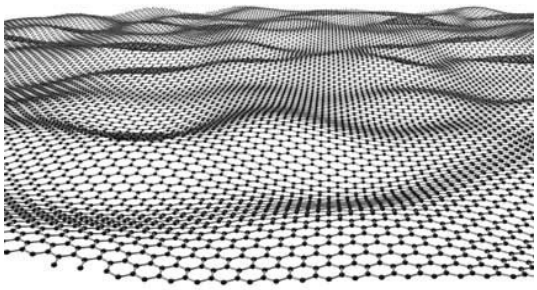
The context : The general context of this Internship/PhD is the study of **membranes**. Membranes occur in several area of physics : in soft matter physics, chemical physics, condensed matter physics, biology, and even in high-energy physics. According to their internal structure one distinguishes two main categories of membranes : **fluid membranes** where the molecules freely diffuse inside the plane of the membrane and **polymerized membranes** where the molecules strongly interact and form a rigid network. This last kind of membranes – the most interesting ones from the point of view of phase transitions – is encountered in biology (e.g. red blood cells) and in condensed matter physics since new materials like **graphene** are, as for their mechanical properties, extremely well described by two dimensional polymerized membranes.

Membranes thus lie at the crossroad of many domains of physics with the common goal to understand the amazing, extremely various and sometimes very complex behaviours that emerge from the interplay between (**thermal-, quantum- or disorder-**) fluctuations and **geometry** of two dimensional systems.



Red blood cell



Graphene sheet

The method : The last twenty years have seen the emergence of efficient tools able to deal with these strongly fluctuating complex systems. Among these tools the **nonperturbative renormalization group** is a particularly powerful one. The renormalization group is a general tool to capture the effects of fluctuations in many body systems. However, up to recent years, this framework operated only perturbatively *i.e.* calling for a – small – expansion parameter (coupling constant, temperature, etc). As a consequence the validity of the predictions so far obtained where restricted to finite domains of physical parameters (coupling constants) or of dimensions. For instance only four dimensional membranes were safely described while one is obviously interested by two dimensional ones ! A nonperturbative formulation of the renormalization group has been formulated in the 90's. It is essentially based on the evolution equation of a running running (Gibbs) effective action $\boldsymbol{\Gamma_k}$ in which high momentum fluctuations – $\boldsymbol{k \leq q \leq \Lambda}$ – (where Λ is a high momentum scale, typically the inverse of the lattice space a) have been integrated out. One thus has :

$\boldsymbol{\Gamma_k = H[\phi]}$ at the microscopic scale $\boldsymbol{k = \Lambda}$

$\boldsymbol{\Gamma_k = \Gamma_k[\phi]}$ at a generic running scale $\boldsymbol{k}$

$\boldsymbol{\Gamma_k = \Gamma[\phi]}$ at the macroscopic scale $\boldsymbol{k = 0}$

$\boldsymbol{\Gamma_k}$ thus interpolates between the microscopic Hamiltonian $\boldsymbol{H}$ and the usual Gibbs free energy $\boldsymbol{\Gamma}$

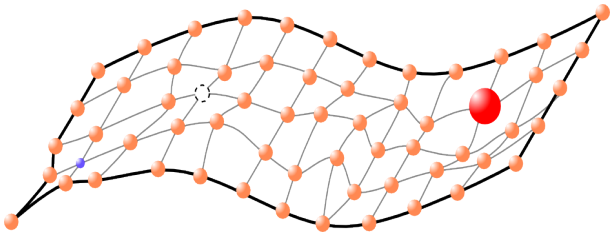
The remarkable fact is that $\boldsymbol{\Gamma_k[\phi]}$ follows an **exact equation** :

$$\boxed{\partial_k \Gamma_k[\phi] = \frac{1}{2}\int d^d q\ \partial_k R_k(q^2)\ \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(q^2)}}$$

where $\boldsymbol{R_k(q^2)}$ is a function of the momenta that separates the high ($\boldsymbol{k \leq q \leq \Lambda}$) and low ($\boldsymbol{0 \leq q \leq k}$) momenta degrees of freedom.

This equation, being nonperturbative in the coupling constant and in the dimension, allows to investigate a large spectrum of situations including in particular those involving large coupling constants and large deviations from the upper critical dimension $D = 4$. For this reason it is now possible to describe in a satisfying way realistic biological structures like vesicles or materials like graphene and similar compounds.

The project : Our goal here is to get a better understanding of various physical aspects of membranes and notably of those of membranes submitted to **disorder** that relies, for instance, on the presence of **random heterogeneities** in the network of polymerized membranes.



Random heterogeneities in a polymerized membrane

Disorder is known to possibly give rise to **new universality classes**. This is, for instance, the case for magnetic systems where disorder is known to lead to new critical exponents and new phase transitions. Recent computations based on the NPRG have precisely led to predict a **new phase transition** and a **new "glassy" phase** controlled by disorder in polymerized membranes (O. Coquand, K. Essafi, J.-P. Kownacki and D. Mouhanna **Phys. Rev. E. 97, 030102 (2018)** and **Phys. Rev. E 101, 042602 (2020)**).

The main aims of this Internship/PhD are, among others : 1) to clarify the **relation between the perturbative and nonperturbative** RG approaches being given that the novel features are not reachable within perturbation theory at low orders 2) to investigate the effects of **quantum fluctuations** on disordered membranes 3) to understand the effects of disorder on the **electronic degrees of freedom** of graphene sheets 4) to investigate the physics of **out-of-equilibrium** membranes.

References

• Membranes : **D. R. Nelson, T. Piran, and S. Weinberg, eds., Proceedings of the Fifth Jerusalem Winter School for Theoretical Physics (World Scientific, Singapore, 2004), 2nd ed.**

• Nonperturbative renormalization group : **B. Delamotte, An introduction to the nonperturbative renormalization group, https ://arxiv.org/pdf/cond-mat/0702365.pdf**

• Perturbative renormalization group applied to membranes : **D. Mouhanna** *et al.* **Phys. Rev. E 101, 062104 (2020)**

• Nonperturbative renormalization group applied to membranes : **D. Mouhanna** *et al.* ***Phys. Rev. E.* 79, 040101 (2009), *Phys. Rev. Lett.* 106, 128102 (2011), *Phys. Rev. E.* 89, 042101 (2014), *Phys. Rev. E* 94, 032125 (2016), *Phys. Rev. E.* 97, 030102 (2018), *Phys. Rev. E* 101, 042602 (2020)**