Internship / Thesis topic:
Noisy quantum walks at the era of NISQ devices

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Quantum walks (QWs) can be used theoretically as discretizations of the Dirac equation, which describes the motion of relativistic quantum particles of matter. In practice, QWs can be implemented with quantum technologies and hence be used as actual quantum simulators (i.e., specific-task, possibly analog rather than digital, quantum computers) for, in particular, the simulation of fundamental-physics dynamics. Currently, however only NISQ (noisy intermediate scale quantum) computers are available – and this still for some time ahead of us – , and so we ought to study the effect of noise on these machines. It is known that the effect of temporal noise on QWs leads to diffusion, while the effect of spatial noise leads to localization, and combining both leads to diffusion. However, there is still work to be done on the continuum-limit descriptions of such noisy models, on short, mid, and long time scales, and this is the purpose of the present proposal. We expect to find models of diffusion, but also of noisy Dirac equations (see Ref. [1] for a first study on these).

I. EXTENDED ABSTRACT

A. A fundamental-physics-oriented introduction to quantum walks

Quantum walks (QWs) are models of quantum transport on a spacetime lattice. There are two classes of QWs: discrete-time (DQWs), and continuous-time ones. We focus on DQWs. The two fundamental properties of DQWs are the following: (i) unitarity, i.e., "quantumness" – which is not specific to discrete-time QWs but is also present in continuous-time ones –, and (ii) locality, which is specific to discrete-time QWs and corresponds to the fact that these DQWs are (quantum) automata, that is, the state of the walker at time $t + \epsilon$ and position x only depends on the states which at time t are within a bounded spatial neighborhood around x, and the size of this neighborhood is fixed.

This "locality" property could also be called "relativistic locality", in the sense that there is an upper bound on the speed at which information can propagate in the model. Actually, the relationship between DQWs and relativistic physics is truly fundamental: DQWs can be used to discretize the Dirac equation, which describes the motion of relativistic particles of matter. DQWs, which as we have seen are conceptually fundamental as discretizations of fundamental-physics equations of motion, can be implemented with quantum technologies [2], i.e., they can serve as tools for quantum simulation, that is, simulating a quantum system with another quantum system in order to make the simulation tractable (no exponential growth of the number of degrees of freedom).

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B. Noise in quantum walks: what is known

Currently, however – and still for some years ahead of us – the only quantum simulators or small quantum computers to which we have access are quite noisy, and called NISQ (noisy intermediate-scale quantum) computers. This is one reason to be interested in the effect of noise on quantum walks. While DQWs have – as the Dirac equation –, a propagative, i.e., ballistic behavior, that is to say, the spread of the distribution is linear in the time t, noise has a non trivial effect on DQWs, which has been studied numerically, and also analytically [3]. The results are essentially the following: at long times, temporal noise leads to diffusion, spatial noise leads to localization, and the addition of temporal and spatial noise to diffusion [3].

We are interested in the continuum-limit descriptions of noisy DQWs (formally, the continuum limit can be explored by taking the spacetime-lattice step going to zero). A first study has been done for *temporal* noise [1], which echoes results from Refs. [4, 5]. Two directions can be taken, both of which involve doing numerical simulations to guide analytical studies. The first direction that can be taken is to deepen our understanding of the results of Ref. [1], so it would deal with continuum limits of DQWs under *temporal* noise. The second direction that can be taken is to investigate continuum limits of DQWs under *spatial* noise.

C. Noise in quantum walks: what we are interested in

1. *First direction: continuum limit of temporally noisy DQWs*

In Ref. [1], we find a certain continuum limit for temporally noisy DQWs. More precisely, we show the numerical consistency of a temporally noisy DQW to simulate a

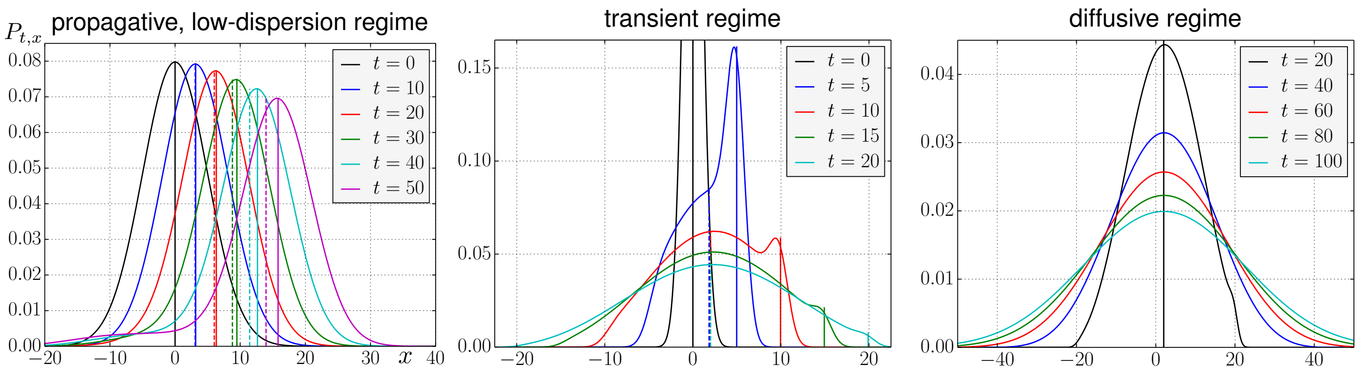

Figure 1. Evolution of a wavepacket via a Lindblad equation with Dirac-equation Hamiltonian part and temporal noise.

Lindblad equation (partial differential equation typical of noisy quantum systems). This Lindblad equation yields, phenomenally, a quantum relativistic diffusion, where the quantum particle first propagates (spread linearly dependent on time) and then diffuses (spread depending on the square root of time), see Fig. 1. Numerical consistency means the following: the error committed by the discretization at the level of the solution of the equation, between one time instant and the next one, goes to zero with the time step. Convergence has not been proven. There are serveral notions of convergence. Observational convergence means the following: the error committed between one time instant and an arbitrarily far other time instant, goes to zero with the time step. It is not guaranteed that any convergence to the Lindblad equation actually holds, since, e.g., at long times we know that we have diffusion so some convergence may be provable only towards a diffusion equation. The convergence properties of the scheme also depend on the initial conditions. These are the questions that we will explore. Notice that the observational convergence of free DQWs towards the Dirac equation has been proven [6].

2. *Second direction: continuum limit of spatially noisy DQWs*

Regarding this second direction, the difficulties are evoked in Ref. [1]. Basically, it is not at all guaranteed that any analytical continuum limit can be found if we introduce spatial noise, but in any case numerical studies will guide us in this research.

3. *Tools to study noisy quantum walks*

When a quantum system is submitted to noise, the state of the system is not anymore described by a state vector in a Hilbert space, but by a density matrix, to which we can associate a Wigner function. Our group has already proved central-limit theorems for temporally noisy QWs described by density matrices [7], and suggested two different definitions of Wigner functions [8, 9], which will be reviewed and used.

II. SUPERVISORS

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