Data assimilation with Physics Informed Neural Networks for Geophysical applications

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Data assimilation looks at characterizing the state $\mathbf{x}$ of a complex system, given several incomplete observations $\mathbf{y}$. A simple formulation of the problem is:

$$\mathbf{y} = H(\mathbf{x}) + \epsilon \tag{1}$$

where H is the operator linking $\mathbf{x}$ and $\mathbf{y}$, and ϵ models the error in H. Multiple methods, such as Kalman filter, Particle filter, variational approaches or Deep Learning ones exist to solve this inverse problem [1]. However, most of these methods require a pre-defined mathematical model of the H operator, or have definitions of H that are fully black boxes.

Physics Informed Neural Networks (PINNs) [2] are deep learning models containing (physic-based) differential equations in the loss function. In this internship, we aim at developing a PINN model for data assimilation:

$$\mathbf{x} = \Psi_{\text{PINN}}(\mathbf{y}) \tag{2}$$

where Ψ_{PINN} is a learned Neural Network, but with a loss function that is informed by pre-defined physical laws. Moreover, these laws can contain learned parameters to increase their flexibility and/or help in the characterization of the state of the system.

This internship has two main objectives:

1. compare the performances of several state of the art data assimilation methods when dealing with multiscale complex systems such as turbulence or atmospheric dynamics.
2. introduce a data assimilation scheme based on PINNs, and compare it against the state of the art.

This study will be performed on two multiscale non-linear dynamical models, for which the state $\mathbf{x}$ can be fully known. We will study a shell model of turbulence [3] and the Lorenz-96 model of the atmosphere [4].

The shell model of turbulence is described by the following set of equations:

$$\frac{du_n}{dt} = i(ak_{n+1}u_{n+2}u^*_{n+1} + bk_n u_{n+1}u^*_{n-1} - ck_{n-1}u_{n-1}u_{n-2}) - \nu k_n^2 u_n + f_n \tag{3}$$

for $n = 1, 2, 3, ...$, u_n being a Fourier component of the velocity field, associated to the wave numbers k_n. These are taken to be $k_n = k_0\lambda^n$, with $\lambda > 1$ being the shell spacing parameter, and $k_0 > 0$.

The Lorenz-96 model is described by the following set of equations:

$$\frac{dx_n}{dt} = (x_{n+1} - x_{n-2})x_{n-1} - x_n + F \tag{4}$$

for $n = 1, 2, 3, ...$, x_n being a value of some atmospheric quantity in a sector of a latitude circle, and F being a forcing.

On these specific study cases, the considered observations $\mathbf{y}$ will be the large scale dynamics of the system together with some local observations of the small scale dynamics. The aim is to recover the full state $\mathbf{x}$ containing all the dynamics. Figure 1 presents a visual description of $\mathbf{y}$ and $\mathbf{x}$.

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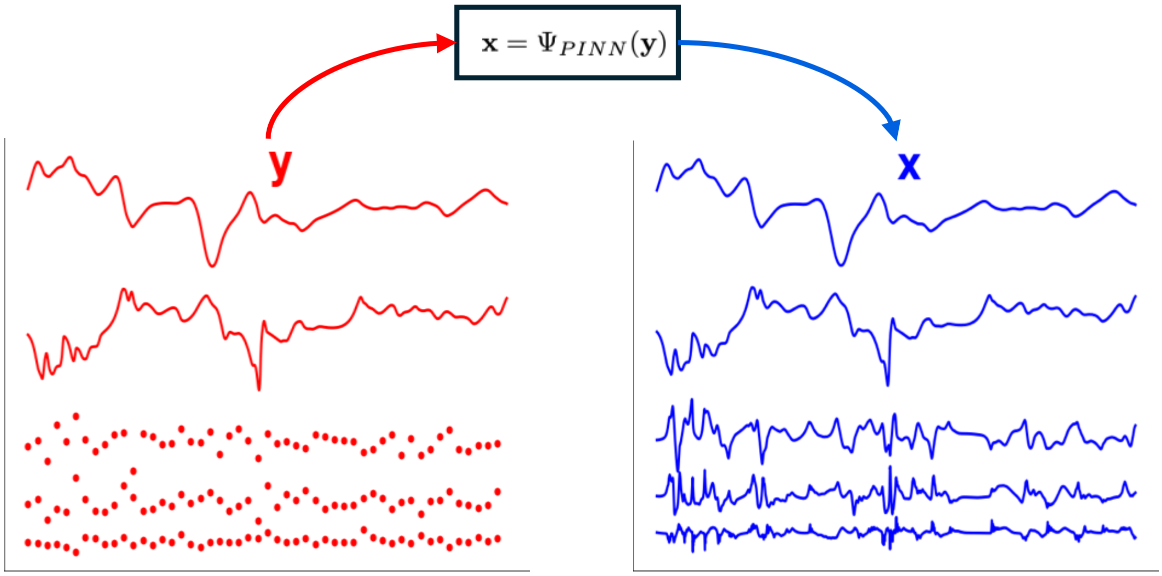

Figure 1: Visual representation of the observation $\mathbf{y}$ (in red) and the full state of the system $\mathbf{x}$ (in blue).

The study is interdisciplinary with an interplay of applied mathematics, fluid physics, geophysics and informatics. This internship is part of a larger project in collaboration with the LISN and INRIA at Paris Saclay (Cyril Furtlehner and Sergio Chibbaro), and it can be followed by a Phd thesis on different axis. Depending on the skills of the candidate, different tracks can be explored.

Motivated students should send a CV and a motivation letter to : carlos.granero-belinchon@imt-atlantique.fr.

References

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